ElCoM

ELECTRICAL COMPUTER MECHATRONICS

تقدم لجنة

ملخص لمادة:

إحصاء و احتمالات

جزيل الشكر للطالب:

حمزة اسماعيل

* Probability and statistics :-

* Ch1 :- Introduction to statistics :-

* Relationship between Probability and inferential statistics :-

1- Population :- المجتمع الاحصائي
⇒ contains all elements .

2- Sample :- العينة الاحصائية
⇒ Part of Population .

* Data Analysis :-

Measures	Population	Sample
1- Mean الوسط الحسابي	$m = \frac{\Sigma X}{n}$	$\bar{X} = \frac{\Sigma X}{n}$
2- Median الوسيط	med متوسط القيم	med متوسط القيم
3- Mode المنوال	الرقم الأكثر تكراراً	الرقم الأكثر تكراراً
4- Rang المدى	R = max - min	R = max - min
5- Variance التباين	$\sigma^2 = \frac{\Sigma(X-\bar{X})^2}{n}$	$S^2 = \frac{\Sigma(X-\bar{X})^2}{n-1}$
6- standard deviation	$\sigma = \sqrt{\frac{\Sigma(X-\bar{X})^2}{n}}$	$S = \sqrt{\frac{\Sigma(X-\bar{X})^2}{n-1}}$

* Graphical Diagnostic :-

1. scatter Plot :- الرسم المبعثر
2. stem and Leaf Plot .
   - ↳ single stem and leaf Plot .
   - ↳ double stem and leaf Plot .
3. Histogram Plot .

* mean of Histogram $= \frac{\Sigma(\text{midPoint} * \text{freq})}{\Sigma f}$
$= \Sigma(\text{midPoint} * \text{Rel fre})$

* Relative freq $= \frac{\text{freq}}{\Sigma f}$.

* Graphical Diagnostic :-

4- Box Plot :-

first Quartile (Q_1) and third Quartile (Q_3)

[1] location of $Q_1 = 0.25(n+1)$.
[2] location of $Q_2 = 0.5(n+1)$.
[3] location of $Q_3 = 0.75(n+1)$.

$\boxed{Q_1 = 2.75 \quad Q_3 = 8.25}$

[1] Value of Q_1 = Value of location (2) $+ 0.75(V.loc(3) - V.loc(2))$.

[2] Value of Q_3 = Value of location (8) $+ 0.25(V.loc(9) - V.loc(8))$.

* Inter-Quartile Rang (IQR) :-

$IQR = Q_3 - Q_1$

lower out lier $= Q_1 - 1.5\ IQR$.
upper out lier $= Q_3 + 1.5\ IQR$.

* Ch2 :- Probability :-

* Sample space (S) :- الفضاء العيني
⇒ The set of all Possible out comes .

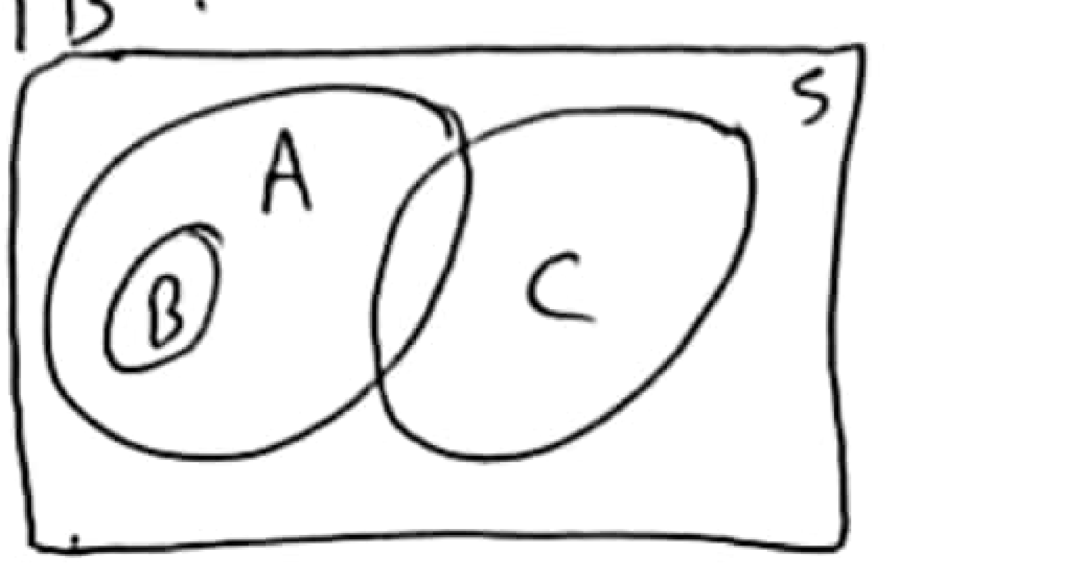
* The Union (∪) and the Intersection (∩)

- Union (∪ الاتحاد) ⇒ The event that contain all elements of the group(A) and group(B)
- Intersection (∩ التقاطع) ⇒ العناصر المشتركة
- Mutually exclusive (disjoint) :- الحوادث المنفصلة

$A \cap B = \Phi$. A, B haven't element in common

* Venn diagram :- اشكال فين

$(A \cap \Phi) = \Phi$ تقاطع
$(A \cup \Phi) = A$ اتحاد
$(A \cup A') = S$ اتحاد $A' \to A$ complement
$S' = \Phi$. $\Phi' = S$.
$(A')' = A$.
$(A \cap B)' = A' \cup B'$
$(A \cup B)' = A' \cap B'$.



* Probability and statistics :-

* Ch2 :- Probability :-

* A Permutation :- . التباديل

an arrangment of all or Part of a set of objects.

1- Linear Permutation :-

[1] The number of Permutation of (n) Object is $\boxed{n!}$.

[2] The number of Permutation of (n) distinct object taken (r) at a time is $\boxed{P_r = \frac{n!}{(n-r)!}}$.

[3] The number of Permutation of (n) object where (n_1) repeated items, (n_2) repeated items :

$$\boxed{P = \frac{n!}{n_1!\ n_2! - n_k!}}$$

2- Circular Permutation:-

[1] The number of Permutation of (n) objects arranged in a circle is $\boxed{(n-1)!}$: without reference Point.

[2] The number of Permutation of (n) objects with reference Point $\boxed{n!}$.

* Combinations :-

The number of combinations of (n) distinct object taken (r) at a time. is

$$\boxed{\binom{n}{r} = \frac{n!}{r!\,(n-r)!}}$$

* arranged ⇒ Permutation

distinguish ⇒ Permutation.

and ⇒ Intersection تقاطع

or ⇒ Union اتحاد

both ⇒ Intersection تقاطع

* The Probability :-

A Propability of an event (A) is the sum of the weight of all sample Points in (A) ⇒ $0 \le P(A) \le 1$.

$A_1, A_2, A_3, \dots$ mutually exclusive events

$$P(A_1 \cup A_2 \cup A_3 \dots) = P(A_1) + P(A_2) + P(A_3)$$

* Theorems :-

1- the Probability of event (A) is

$$P(A) = \frac{n}{N} \quad \begin{matrix} \to \text{number of events in (A)} \\ \to \text{number of sample space.} \end{matrix}$$

2- If (A) and (B) are two events then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

اتحاد . تقاطع .

3- If (A) and (B) mutually exclusive

$$P(A \cup B) = P(A) + P(B).$$

4- If A_1, A_2, A_3 – mutually exclusive

$$P(A_1 \cup A_2 \cup A_3 \dots) = P(A_1) + P(A_2) + P(A_3)$$

5- for three events A. B and C :-

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) + P(A \cap B \cap C) - P(A \cap B) - P(A \cap C) - P(B \cap C).$$

6- If (A) and (A') are complementary event

$$P(A) + P(A') = 1$$

$$(A \cup A') = S \Rightarrow P(S) = 1.$$

7- The conditional probability of (B) given (A) denoted by $P(B/A)$.

$$P(B/A) = \frac{P(A \cap B)}{P(A)}.$$

8- Two event (A),(B) are independent

$$P(B/A) = P(B) \quad . \quad P(A/B) = P(A).$$

9. the event (A) and (B) both occur

$$P(A \cap B) = P(A)\,P(B/A).$$

$$P(A \cap B) = P(A) * P(B).$$

* Probability and statistiss :-

* Ch3 :- Random Variable and Probability distributions :-

* Random Variable :- المتغير العشوائي
is a function that associates areal number with each element in the Sample space. (S).

* Discrete sample space and the Continuous sample space :-

Discrete Sample space → Table.

Continuous Sample space → function دالة

* Discrete Random Variable :-
Probability mass function or Probability of the discrete random Variable if

1. $f(x) \geq 0$.   2. $\sum_{x} f(x) = 1$.

3. $P(X=x) = f(x)$.

* The Cumulative distribution function of a discrete random Variable :-

$$F(x) = \boxed{P(X \leq x)} = \sum_{t \leq x} f(t) \quad -\infty < x < \infty$$

* Continuous Probability Distributions :-
Typical density function

$$P(a<X<b) = \int_a^b f(x)\,dx$$

* Continuous Probability Distributions
Probability density function for the Continuous random Variable (X) if.

1. $f(x) \geq 0$, 2. $\int_{-\infty}^{\infty} f(x)\,dx = 1$

3. $P(a<x<b) = \int_a^b f(x)\,dx$.

* The Cumulative distribution function of a Continuous random Variable with density function $f(x)$ is :-

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx \quad -\infty < x < \infty$$

$$P(a<x<b) = F(b) - F(a).$$

* Joint Probability distribution or Probability mass function of the discrete random Variable (X) and (Y) if

1. $f(x,y) \geq 0$. 2. $\sum_{x}\sum_{y} f(x,y) = 1$

3. $P(X=x \,.\, Y=y) = f(x,y)$

* Joint density function of the continuous random Variable (X) and (Y) if.

1. $f(x,y) \geq 0$. 2. $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$

3. $P((x,y) \in A) = \iint_A f(x,y)\,dx\,dy$.

* The marginal distributions of (X, Y) :-

- marginal distributions for the discrete

$$g(x) = \sum_{y} f(x,y) \qquad h(y) = \sum_{x} f(x,y).$$

- marginal distributions for continuous

$$g(x) = \int_{-\infty}^{\infty} f(x,y)\,dy \qquad h(y) = \int_{-\infty}^{\infty} f(x,y)\,dx.$$

* The Conditional distribution of (X, Y) :-

conditional distribution of (Y) given that (X)

$$f(y/x) = \frac{f(x,y)}{g(x)} \,. \quad g(x) > 0.$$

conditional distribution of (X) given that (Y)

$$f(x/y) = \frac{f(x,y)}{h(y)}$$

independent $f(x,y) = g(x)\,h(y)$.

Probability and statistics :-

Ch4 :- Mathematical Expectation :-

* let $\underline{X,y}$ be arandom Variable with Probability distribution $f(x)$
Then the Mean or expected Value $\underline{(of\ x)}$
if (X) is discrete . $M = E(x) = \sum x f(x)$
if (X) is continuouse $M = E(x) = \int_{-\infty}^{\infty} x f(x) dx$

the mean of $g(x)$.
if $g(x)$ is discrete $M = E(g(x)) = \sum g(x) f(x)$
if $g(x)$ is continuous $M = E(g(x)) = \int_{-\infty}^{\infty} g(x) f(x) dx$
the mean of Joint function of $g(x,y)$
if $g(x,y)$ is discrete $E(g(x,y)) = \sum_x \sum_y g(x,y) f(x,y)$
if $g(x,y)$ is continuous $E(g(x,y)) = \int_x \int_y g(x,y) f(x,y) dx dy$

* let $\underline{X,y}$ be a random Variable with Probability distribution $f(x)$ and mean (M) the Variance of (X) is
if (X) is discrete $\sigma^2 = E[(X-M)^2] = \sum (x-M)^2 f(x)$
if X is continuous $\sigma^2 = E[(x-M)^2] = \int_{-\infty}^{\infty} (x-M)^2 f(x) dx$
$\sigma_X^2 = \underline{E(X^2)} - M_X^2 \Rightarrow E(x^2) = \sum x^2 f(x)$ $M = \sum x f(x)$
$\sigma_X^2 = E(x^2) - M_x^2 \Rightarrow E(x^2) = \int x^2 f(x)$ $M = \int x f(x)$

* Means of Linear Combination
$M_{(ax+by+c)} = a M_x + b M_y + C$.
$M_{(2x-1)} = 2 M_x - 1$.
Variance of linear combination
$\sigma^2_{(ax+by+c)} = a^2 \sigma_X^2 + b^2 \sigma_y^2 + 2ab\ \sigma_{xy}$.
$\sigma^2_{(ax+by)} = a^2 \sigma_X^2 + b^2 \sigma_y^2$
$\sigma^2_{(ax-by)} = a^2 \sigma_x^2 + b^2 \sigma_y^2$

* Variance of $g(x)$
$\sigma^2_{g(x)} = E(g(x)^2) - M^2_{g(x)}$.
discrete $\Rightarrow E(g(x)^2) = \sum g(x)^2 f(x)$.
continuous $\Rightarrow E(g(x)^2) = \int g(x)^2 f(x) dx$.

* Covarince of (xy):
$\sigma_{xy} = E(\underline{xy}) - M_x M_y$.
discrete $\Rightarrow E(xy) = \sum\sum xy\ f(x,y)$
continuous $\Rightarrow E(xy) = \int_x \int_y xy\ f(x,y)\ dx dy$.

if X,y are independent then
$E(xy) = E_x E_y$.
$\Rightarrow \sigma_{xy} = Zero$.

* Correlation cofficents :-
$\rho_{xy} = \dfrac{\sigma_{xy}}{\sigma_x \sigma_y}$

$\sigma_{xy} = \rho_{xy}\ \sigma_x \sigma_y$.

$E(xy) - M_x M_y = \rho_{xy}\ \sigma_x \sigma_y$.

$M_x = g(x)\ f(x,y)$.
$M_y = h(y)\ f(x,y)$.

*Probability and statistics :-

*Ch5: Discrete Uniform Distribution :-

*Bionomial Distribution :- خو الحدين .

⇐ مجموع احتمالين = واحد.

$$B(X,n,P) = \binom{n}{X} P^{X} q^{n-X}$$

$$P+q=1 \Rightarrow \boxed{q=1-P}.$$

X ← العدد المطلوب إيجاد ال (P) ، n ← عدد مرات تكرار التجربة .

P ← احتمالية وقوع الحدث مرة واحدة .

$B(X=a) = B(a) - B(a-1)$.

$B(X \leq a) = B(a)$.

$B(X<a) = B(a-1)$.

$B(X \geq a) = 1 - B(a-1)$.

$B(X>a) = 1 - B(a)$.

$P(1 \leq X < 5) = B(4) - B(0)$.

1, 2, 3, 4

$P(2 < X < a) = B(8) - B(2)$.

3, 4, 5, 6, 7, 8

$Mean = nP \qquad \sigma^2 = nPq$.

ملاحظات :-

n ← أكبر رقم بالسؤال   P ← $(0 \leq P \leq 1)$.

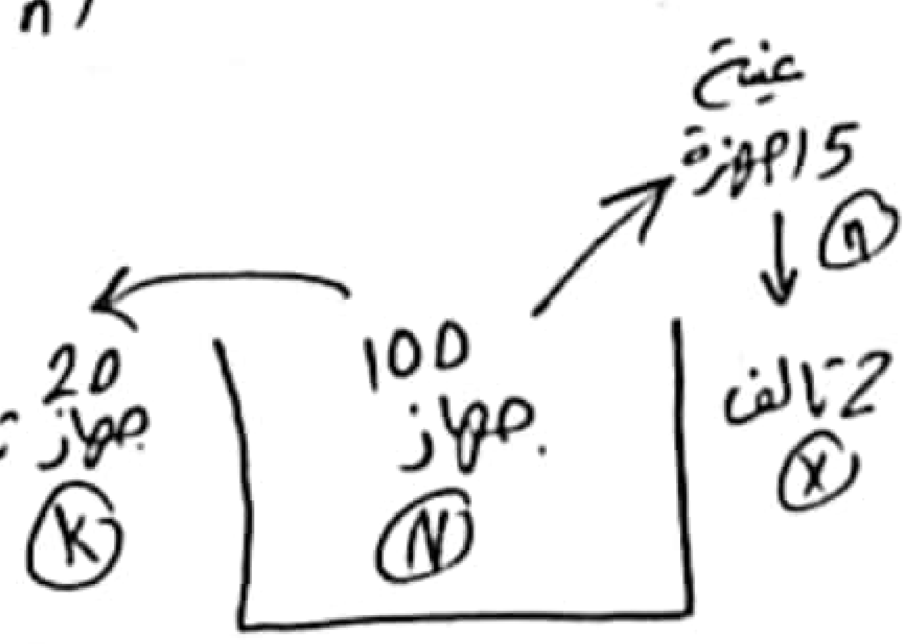
at least ⇒ ≥ أكبر ، atmost ⇒ ≤ أقل

*Hyper geomtric :- فيها اربع أرقام.

$$H(X,N,n,K) = \frac{\binom{K}{X}\binom{N-K}{n-X}}{\binom{N}{n}} \qquad n<N, \; X<K.$$

$$mean = \frac{nK}{N}$$

$$\sigma^2 = \frac{N-n}{N-1} \times n \times \frac{K}{N} \times \left(1-\frac{K}{N}\right).$$



* ملاحظات :-

N ← العينة الكبيرة (أكبر رقم).

n ← العينة المأخوذة من العينة ~~الصغيرة~~ الكبيرة.

K ← عدد ال deflective بالعينة الكبيرة.

X ← احتمالية وجود ال deflect في العينة الصغيرة.

* حالة خاصة ومهمة :-

$$\boxed{\frac{n}{N} \leq 0.05} \rightarrow \text{Bionomial Distribution}$$

Hyper → Bionomial.

N = 1000, n = 5, X = 2, K = 100

n = 5, X = 2, $P = \frac{100}{1000} = 0.1$.

*Negative and geometric distribution :-

$$b(X,K,P) = \binom{X-1}{K-1} P^{K} q^{X-K}.$$

K ← متى اول مرة تحقق هذه التجربة نجاحها.

X ← احتمال النجاح من (n) كلها.

$$mean = \frac{K}{P} \qquad \sigma^2 = \frac{Kq}{P}.$$

*geometric distribution :-

هو حالة خاصة من ال negative . بيكون فيها $\boxed{K=1}$

$$b(X,1,P) = \binom{X-1}{1-1} P^{1} q^{X-1}$$

$$g(X,P) = P\, q^{X-1}.$$

$$P+q=1 \Rightarrow \boxed{q=1-P}$$

*Poisson Distribution :-

$$P(X,\lambda) = \frac{e^{-\lambda t} (\lambda t)^{X}}{X!}$$

نستخدم جداول ال Poisson بنفس طريقة ال Bionom.

$P(X=a) = P(a) - P(a-1)$.

$P(X \geq a) = 1 - P(a-1)$.

$P(X \leq a) = P(a)$

$P(X>a) = 1 - P(a)$

$P(X<a) = P(a-1)$

$*P(2<X<6) = P(5) - P(2)$.

$*P(3 \leq X \leq 7) = P(7) - P(2)$.